Coupled Nonuniform Transmission Line and Its Applications

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Abstract—**Theory and applications of coupled nonuniform transmission lines are described. Matrix representations of a general coupled nonuniform transmission line are presented, by means of which the behavior of any coupled nonuniform transmission line may be completely described. Among a wide variety of applications of coupled nonuniform transmission lines, two typical networks, one the coupled nonuniform transmission-line folded all-pass network and the other the coupled nonuniform transmission-line directional coupler, are treated in detail. Equivalent circuit representations of these two networks are presented, which enable the designer to synthesize them in a greatly simplified manner by making use of the theories now available for more conventional single nonuniform transmission lines. In addition, the properties of these two networks using coupled exponential line are investigated. Design procedure is also given for asymmetrical coupled exponential-line directional couplers having excellent characteristics.**

I. Introduction

THERE HAS BEEN a considerable amount of work reported in Kaufman [1] concerned with various types of nonuniform transmission lines. However, the majority refers to the single nonuniform transmission line and the available theory does not apply to coupled nonuniform transmission lines. These may have a wide variety of applications in the field of distributed constant networks because of the numerous advantages of the parallel coupling effect and, in addition, may offer the possibility of the realization of a class of ultra-broadband components from the nature of nonuniformity. Therefore, it would be highly desirable to investigate coupled nonuniform transmission lines as distributed network elements.

The work described in this paper is presented in the following manner: in Section II an improved method of analysis of coupled nonuniform transmission lines is presented, which allows the four-port matrix parameters of such coupled transmission lines to be obtained in a very concise manner. Remarkably simple formulas for various four-port matrix representations are derived. In Section III, among numerous applications of coupled nonuniform transmission lines, the folded all-pass network and the coupled nonuniform transmission-line directional coupler are treated in detail. Theory of the preceding section is applied and the equivalent circuit representations of these two networks are presented, which may reduce the syntheses of these networks to those of single nonuniform transmission lines. In Section IV, the phase and delay characteristics of the coupled nonuniform transmission-line folded all-pass networks, and the coupling characteristics of the coupled nonuniform transmission-line directional couplers are investigated, taking the coupled exponential line as the network element.

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II. Matrix Representations of Coupled Nonuniform Transmission Lines

The coupled nonuniform transmission line to be considered in this paper is a symmetrical two-conductor line with common return, in which the line parameters vary along the longitudinal direction, that is, along the direction of propagation of electromagnetic waves. A convenient way to describe the behavior of such a coupled transmission line is by means of the various matrix representations; i.e., impedance matrix, admittance matrix, etc., which will be derived in this section. One derivation method for the matrix representation of coupled nonuniform transmission lines utilizes coupled uniform transmission-line techniques. For example, transfer matrix may be derived by:

1) Dividing the coupled nonuniform transmission line of finite length into n elementary sections of identical length Δl.
2) Multiplying the transfer matrices of n elementary sections, each of which is approximated by the coupled uniform transmission line section.
3) Taking the limit for $n \to \infty$ and $\Delta l \to 0$, keeping the total length constant.

This method, however, is tedious to apply and, furthermore, the transformation from the transfer matrix to the other matrix representations becomes intractable. Hence, this paper employs an alternative method which seems to have the advantage of greater simplicity.

Let us consider now the coupled nonuniform transmission-line four-port composed of two conductors of identical length l, having reflection symmetry to one another about a longitudinal axis, as illustrated in Fig. 1. The derivation method for the matrix representations adopted consists of the usual procedure of reducing the problem of a four-port network to that of a two-port network by taking advantage of the symmetry about the plane T-T'. The behavior of such a coupled nonuniform transmission-line four-port can be completely described by superposition of the two fundamental modes, so-called even and odd modes [2], [3]. In the even mode, for which the respective voltages and currents on the two conductors are equal and of the same sign, the plane of symmetry may be replaced by a magnetic wall, while in the odd mode, for which the respective voltages and currents are equal but of opposite sign, this may be replaced by an electric wall at zero potential. Throughout this paper, sub- or superscripts e and o refer to the even and odd modes, respectively.

We shall first derive the impedance matrix, for which the fundamental modes are excited by the sets of constant current generators as shown in Fig. 2(a) for even mode and 2(b) for odd mode. Any port condition can be expressed as a linear combination of these two modes of excitation. Let the two-port impedance matrices of the even and odd mode half sections of the complete four-port network in Fig. 1 be, writing even and odd mode cases together,[1]

$$[Z^{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.}] = \begin{bmatrix} Z_{11}^{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} & Z_{12}^{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} \\ Z_{21}^{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} & Z_{22}^{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} \end{bmatrix}, \tag{1}$$

where, from the reciprocity condition,

$$Z_{12}^{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} = Z_{21}^{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.}, \tag{2}$$

where the subscripts 1 and 2 denote Ends I and II, respectively (see Fig. 1). By superposition, the impedance matrix of the coupled nonuniform transmission-line four-port in Fig. 1 is found to be

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} Z_{11}^e + Z_{11}^o & Z_{11}^e - Z_{11}^o & Z_{12}^e - Z_{12}^o & Z_{12}^e + Z_{12}^o \\ Z_{11}^e - Z_{11}^o & Z_{11}^e + Z_{11}^o & Z_{12}^e + Z_{12}^o & Z_{12}^e - Z_{12}^o \\ Z_{21}^e - Z_{21}^o & Z_{21}^e + Z_{21}^o & Z_{22}^e + Z_{22}^o & Z_{22}^e - Z_{22}^o \\ Z_{21}^e + Z_{21}^o & Z_{21}^e - Z_{21}^o & Z_{22}^e - Z_{22}^o & Z_{22}^e + Z_{22}^o \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}. \tag{3}$$

On the other hand, if the fundamental modes are excited by the constant voltage generators instead of current generators, manipulation similar to the case of the impedance matrix yields the admittance matrix as

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} Y_{11}^o + Y_{11}^e & Y_{11}^o - Y_{11}^e & Y_{12}^o - Y_{12}^e & Y_{12}^o + Y_{12}^e \\ Y_{11}^o - Y_{11}^e & Y_{11}^o + Y_{11}^e & Y_{12}^o + Y_{12}^e & Y_{12}^o - Y_{12}^e \\ Y_{21}^o - Y_{21}^o & Y_{21}^o + Y_{21}^e & Y_{22}^o + Y_{22}^e & Y_{22}^o - Y_{22}^e \\ Y_{21}^o + Y_{21}^e & Y_{21}^o - Y_{21}^e & Y_{22}^o - Y_{22}^e & Y_{22}^o + Y_{22}^e \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}, \tag{4}$$

where the two-port admittance matrices of the even and odd mode half sections of the complete four-port network in Fig. 1 are assumed such that

$$[Y^{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.}] = \begin{bmatrix} Y_{11}^{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} & Y_{12}^{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} \\ Y_{21}^{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} & Y_{22}^{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} \end{bmatrix}, \tag{5}$$

where, from the reciprocity condition,

$$Y_{12}^{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} = Y_{21}^{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.}. \tag{6}$$

It may be more convenient in some cases to use the transfer matrix which is derived in an analogous manner as

$$\begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} A_e + A_o & A_e - A_o & B_e - B_o & B_e + B_o \\ A_e - A_o & A_e + A_o & B_e + B_o & B_e - B_o \\ C_e - C_o & C_e + C_o & D_e + D_o & D_e - D_o \\ C_e + C_o & C_e - C_o & D_e - D_o & D_e + D_o \end{bmatrix} \cdot \begin{bmatrix} V_4 \\ V_3 \\ -I_4 \\ -I_3 \end{bmatrix}, \tag{7}$$

where the two-port transfer matrices of the even and odd mode half sections are assumed such that

$$[F_{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.}] = \begin{bmatrix} A_{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} & B_{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} \\ C_{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} & D_{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} \end{bmatrix}, \tag{8}$$

where, from the reciprocity condition,

$$A_{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} \cdot D_{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} - B_{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} \cdot C_{\left\{\begin{smallmatrix}e\\o\end{smallmatrix}\right.} = 1. \tag{9}$$

Equations (3), (4), and (7) are remarkably compact formulas which indicate that the matrix representations of a symmetrical coupled nonuniform transmission-line four-port may be readily derived if the two-port matrix parameters of the even and odd mode half sections of the complete four-port network are known. The method presented is directed toward the derivation of the matrix representations of coupled nonuniform transmission lines; however, (3), (4), and (7) may be applicable to general symmetrical four-port networks including not only distributed constant networks but also lumped constant ones.

[1] Equation (1) indicates $[Z^e]$ and $[Z^o]$, simultaneously. Similar representations will be often used in the latter part of this paper.

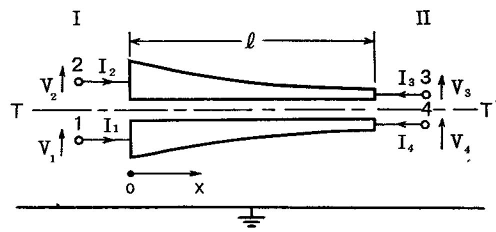

Fig. 1. Coupled nonuniform transmission-line four-port.

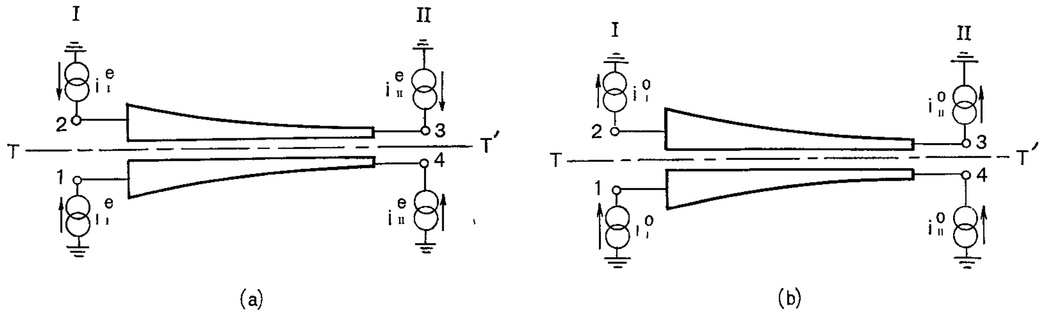

Fig. 2. Excitation of the fundamental modes by means of the constant current generators: (a) even mode, (b) odd mode.

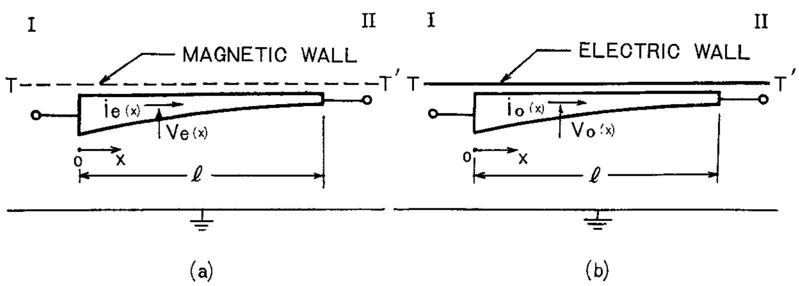

Fig. 3. Even and odd mode half-section single nonuniform transmission-line two-ports: (a) even mode, (b) odd mode.

It now remains to derive the two-port matrix representations of the two single nonuniform transmission lines shown in Fig. 3(a) and 3(b), which correspond to the even and odd mode half sections, respectively, of the coupled nonuniform transmission-line four-port in Fig. 1. Assuming negligible dissipation and TEM propagation for both even and odd mode cases, the even and odd mode propagation constants $\gamma_{\{^e_o}$ identically reduce to

$$\gamma_{\{^e_o} = j\beta = j\frac{2\pi}{\lambda}, \tag{10}$$

where β is the phase constant and λ is the wavelength. Then the differential equations representing the line voltages $v_{\{^e_o}(x)$ and the line currents $i_{\{^e_o}(x)$ for the even and odd mode half-section single nonuniform transmission lines are

$$\frac{d}{dx}\{v_{\{^e_o}(x)\} + j\beta\cdot Z_{0\{^e_o}(x)\cdot i_{\{^e_o}(x) = 0,$$

$$\frac{d}{dx}\{i_{\{^e_o}(x)\} + \frac{j\beta}{Z_{0\{^e_o}(x)}\cdot v_{\{^e_o}(x) = 0, \tag{11}$$

where $Z_{0e}(x)$ and $Z_{0o}(x)$ are, respectively, the even and odd mode characteristic impedances of the half-section single nonuniform transmission line, both being functions of the position x along the line only. Further differentiation of (11) yields the linear second-order differential equations for the even and odd mode line voltages

$$\frac{d^2}{dx^2}\{v_{\{^e_o}(x)\} - \frac{d}{dx}\{\log Z_{0\{^e_o}(x)\}\cdot\frac{d}{dx}\{v_{\{^e_o}(x)\} + \beta^2\cdot v_{\{^e_o}(x) = 0, \tag{12}$$

which may be solved using standard techniques if $Z_{0\{^e_o}(x)$ are known. Let $p_e(x)$ and $q_e(x)$ represent a pair of linearly independent solutions of (12) for even mode case, and, similarly, let $p_o(x)$ and $q_o(x)$ denote a pair of linearly independent solutions of (12) for odd mode case. The general solutions of (12) are given by

$$v_{\{^e_o}(x) = C_{1\{^e_o}\cdot p_{\{^e_o}(x) + C_{2\{^e_o}\cdot q_{\{^e_o}(x), \tag{13}$$

where $C_{1\{^e_o}$ and $C_{2\{^e_o}$ are constants, and the line currents are

$$i_{\{^e_o}(x) = \frac{-1}{j\beta\cdot Z_{0\{^e_o}(x)}[C_{1\{^e_o}\cdot p'_{\{^e_o}(x) + C_{2\{^e_o}\cdot q'_{\{^e_o}(x)], \tag{14}$$

where the prime indicates differentiation with respect to x. The use of (13) and (14) yields the two-port matrix representations of the even and odd mode half sections of a general coupled nonuniform transmission-line four-port in Fig. 1, following the method of Dutta Roy [4],

$$[Z^{\{{}^{e}_{o}}] = \frac{j\beta}{m_2{}^{\{{}^{e}_{o}}} \begin{bmatrix} m_1{}^{\{{}^{e}_{o}} Z_{0\{{}^{e}_{o}}(0) & m_3{}^{\{{}^{e}_{o}} Z_{0\{{}^{e}_{o}}(l) \\ m_4{}^{\{{}^{e}_{o}} Z_{0\{{}^{e}_{o}}(0) & m_5{}^{\{{}^{e}_{o}} Z_{0\{{}^{e}_{o}}(l) \end{bmatrix}, \tag{15}$$

$$[Y^{\{{}^{e}_{o}}] = \frac{1}{j\beta \cdot m_6{}^{\{{}^{e}_{o}}} \begin{bmatrix} -m_5{}^{\{{}^{e}_{o}}/Z_{0\{{}^{e}_{o}}(0) & m_3{}^{\{{}^{e}_{o}}/Z_{0\{{}^{e}_{o}}(0) \\ m_4{}^{\{{}^{e}_{o}}/Z_{0\{{}^{e}_{o}}(l) & -m_1{}^{\{{}^{e}_{o}}/Z_{0\{{}^{e}_{o}}(l) \end{bmatrix}, \tag{16}$$

$$[F_{\{{}^{e}_{o}}] = \frac{1}{m_4{}^{\{{}^{e}_{o}}} \begin{bmatrix} m_1{}^{\{{}^{e}_{o}} & -j\beta \cdot m_6{}^{\{{}^{e}_{o}} \cdot Z_{0\{{}^{e}_{o}}(l) \\ m_2{}^{\{{}^{e}_{o}}/j\beta \cdot Z_{0\{{}^{e}_{o}}(0) & m_5{}^{\{{}^{e}_{o}} \cdot Z_{0\{{}^{e}_{o}}(l)/Z_{0\{{}^{e}_{o}}(0) \end{bmatrix}, \tag{17}$$

where the m's are

$$\begin{aligned}
m_1{}^{\{{}^{e}_{o}} &= p'_{\{{}^{e}_{o}}(l) \cdot q_{\{{}^{e}_{o}}(0) - q'_{\{{}^{e}_{o}}(l) \cdot p_{\{{}^{e}_{o}}(0) \\
m_2{}^{\{{}^{e}_{o}} &= p'_{\{{}^{e}_{o}}(0) \cdot q'_{\{{}^{e}_{o}}(l) - q'_{\{{}^{e}_{o}}(0) \cdot p'_{\{{}^{e}_{o}}(l) \\
m_3{}^{\{{}^{e}_{o}} &= p'_{\{{}^{e}_{o}}(0) \cdot q_{\{{}^{e}_{o}}(0) - q'_{\{{}^{e}_{o}}(0) \cdot p_{\{{}^{e}_{o}}(0) \\
m_4{}^{\{{}^{e}_{o}} &= p'_{\{{}^{e}_{o}}(l) \cdot q_{\{{}^{e}_{o}}(l) - q'_{\{{}^{e}_{o}}(l) \cdot p_{\{{}^{e}_{o}}(l) \\
m_5{}^{\{{}^{e}_{o}} &= p'_{\{{}^{e}_{o}}(0) \cdot q_{\{{}^{e}_{o}}(l) - q'_{\{{}^{e}_{o}}(0) \cdot p_{\{{}^{e}_{o}}(l) \\
m_6{}^{\{{}^{e}_{o}} &= p_{\{{}^{e}_{o}}(0) \cdot q_{\{{}^{e}_{o}}(l) - q_{\{{}^{e}_{o}}(0) \cdot p_{\{{}^{e}_{o}}(l).
\end{aligned} \tag{18}$$

Also, from the nature of the linear second-order differential equation, we get the following relationship:

$$\frac{p'_{\{{}^{e}_{o}}(x) \cdot q_{\{{}^{e}_{o}}(x) - q'_{\{{}^{e}_{o}}(x) \cdot p_{\{{}^{e}_{o}}(x)}{Z_{0\{{}^{e}_{o}}(x)} = \text{constant}. \tag{19}$$

Then it is easily proved that (15), (16), and (17) always satisfy the corresponding reciprocity conditions; i.e., (2), (6), and (9).

Substitution of (15), (16), and (17) in (3), (4), and (7), respectively, yields the impedance, admittance, and transfer matrices of a general coupled nonuniform transmission-line four-port. No preferred matrix representation exists. The matrix representation that is most convenient depends upon the network configuration to be analyzed. It now remains only whether or not (12) is solvable for the given types of variation of the even and odd mode characteristic impedances. Equation (12) is the linear second-order differential equation for single nonuniform transmission lines which, so far, have been investigated by a number of workers in the field [1], [4]–[8]. For the given even and odd mode characteristic impedance functions, the four-port matrix parameters of coupled nonuniform transmission lines can be determined either by direct substitution in (3), (4), and (7), if the two-port matrix parameters of the even and odd mode half-section single nonuniform transmission lines are known, or by the use of (3), (4), and (7) together with (15)–(17), if the solutions of (12) are available.

Although the general solutions of (12) for completely arbitrary $Z_{0e}(x)$ and $Z_{0o}(x)$ have never been accomplished, all the existing solutions for single nonuniform transmission lines are applicable to coupled nonuniform transmission lines. In particular, two methods recently proposed are of importance. One proposed by Berger [9] is a simple generalizing method which enables one to obtain the solutions for single nonuniform transmission lines of various shapes by utilizing those for the previously solved single nonuniform transmission lines. The other method described by Protonotarios and Wing [10], although an approximate one, is applicable to arbitrarily nonuniform transmission lines and is extremely valuable in the case where the closed form solutions are not possible or cannot be found easily.[2] Thus we conclude that it is possible to form a coupled nonuniform transmission line having two single nonuniform transmission lines, for which the solutions of (12) are known, as the even and odd mode half sections. It should be noted, however, that, for the coupled two-conductor line with common return under consideration, physical realizability requires that

$$Z_{0e}(x) \geqq Z_{0o}(x) \qquad (0 \leqq x \leqq l). \tag{20}$$

In the selection of the characteristic impedance functions, (20) must be considered.

III. Applications of Coupled Nonuniform Transmission Lines

There are a number of papers dealing with single nonuniform transmission lines. To date, however, application of this type of transmission line has been limited, from its nature, to a few classes of circuit components such as impedance transformers, resonators, etc. In comparison with such single nonuniform transmission lines, coupled nonuniform transmission lines may have a wide variety of applications in UHF and microwave regions. For example, when the pertinent port conditions are applied to the coupled nonuniform transmission-line four-port, the resultant two-port networks as in Fig. 4 may be used as distributed constant filters which may have sharper cutoff and greatly extended rejection bandwidth than are obtainable with uniform transmission lines, and it is possible to analyze their transmission properties by the use of the matrix representations presented in Section II, if the functional forms of the even and odd mode characteristic impedances are given.

In this section, however, two other networks will be treated in detail. These are the coupled nonuniform transmission-line folded all-pass network and the coupled nonuniform transmission-line directional coupler, both of which possess peculiar characteristics not attainable by conventional single nonuniform transmission lines. Throughout the latter discussion, we treat such a coupled nonuniform transmission line that satisfies the following condition:

$$z_{0e}(x) \cdot z_{0o}(x) = 1, \tag{21}$$

[2] This method is directed toward single nonuniform RC transmission lines; however, it can be easily extended to the case of lossless single nonuniform transmission lines.

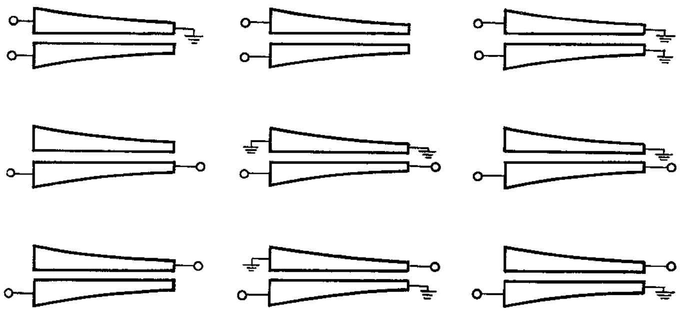

Fig. 4. Coupled nonuniform transmission-line filters.

where $z_{0e}(x)$ and $z_{0o}(x)$ are, respectively, the even and odd mode characteristic impedances normalized to the terminating impedances. As will be shown later, under this condition, the coupled nonuniform transmission-line folded network becomes an all-pass network and the coupled nonuniform transmission-line four-port terminated by unit impedances behaves as a directional coupler with perfect input match and infinite directivity at all frequencies.

It can be easily proved that, under the condition (21), the even and odd mode half sections, shown in Fig. 3(a) and 3(b), respectively, of a coupled nonuniform transmission-line four-port are mutually dual; that is,

$$A_e = D_o, \quad B_e = C_o, \quad C_e = B_o, \quad D_e = A_o, \tag{22}$$

where $A\{^e_o$, $B\{^e_o$, $C\{^e_o$, and $D\{^e_o$ are the two-port transfer matrix parameters of the even and odd mode half section single nonuniform transmission lines.

A. Coupled Nonuniform Transmission-Line Folded All-Pass Network

The coupled nonuniform transmission-line folded network to be analyzed herein is the two-port network shown in Fig. 5, in which two ports at one end of the coupled nonuniform transmission-line four-port are interconnected [2], [11]; ideally this connection should be of zero length. The folded network using a coupled uniform transmission line is known as the microwave *C*-section [12], [13]. We shall now investigate the frequency behavior of the coupled nonuniform transmission-line folded network under the condition (21). If the even mode signals $(+\frac{1}{2}, +\frac{1}{2})$ are applied at Ports 1 and 2, respectively, the plane of symmetry may be replaced by a magnetic wall. Likewise, if the odd mode signals $(+\frac{1}{2}, -\frac{1}{2})$ are applied at Ports 1 and 2, the plane of symmetry may be replaced by an electric wall. In each case, the problem reduces to that of a one-port network, and the sum of these two cases is a single signal of unit amplitude in Port 1. The resultant signals out of Ports 1 and 2 are

$$\begin{aligned} A_1 &= (\Gamma_{0e} + \Gamma_{0o})/2 \\ A_2 &= (\Gamma_{0e} - \Gamma_{0o})/2, \end{aligned} \tag{23}$$

where Γ_{0e} and Γ_{0o} are the reflection coefficients for the even and odd mode half-section single nonuniform transmission-line one-ports, respectively. These are related to the open-circuit impedance z_{op} and short-circuit impedance z_{sh} of half the folded network by

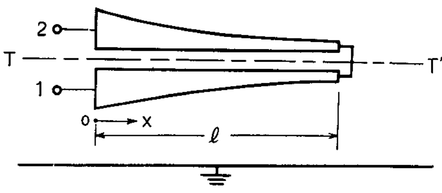

Fig. 5. Coupled nonuniform transmission-line folded network.

$$\begin{aligned} \Gamma_{0e} &= \frac{z_{op} - 1}{z_{op} + 1} = \frac{(A_e/C_e) - 1}{(A_e/C_e) + 1} \\ \Gamma_{0o} &= \frac{z_{sh} - 1}{z_{sh} + 1} = \frac{(B_o/D_o) - 1}{(B_o/D_o) + 1}. \end{aligned} \tag{24}$$

Substitution of (22) into (24) yields

$$\Gamma_{0e} = -\Gamma_{0o}. \tag{25}$$

Then we get from (23) and (25)

$$\begin{aligned} A_1 &= 0 \\ A_2 &= \Gamma_{0e}. \end{aligned} \tag{26}$$

Thus, it is found that the coupled nonuniform transmission-line folded network in Fig. 5 behaves as an all-pass network under the condition (21), and the phase shift ϕ through this all-pass two-port network, after manipulation, is expressed as

$$\begin{aligned} \phi &= \cos^{-1}[\mathrm{Re}(\Gamma_{0e})] \\ &= 2\cdot\tan^{-1}\left[\frac{-1}{\sqrt{\rho(0)}\cdot\beta}\cdot\frac{m_2^e}{m_1^e}\right], \end{aligned} \tag{27}$$

where m_1^e and m_2^e are given by (18) and $\rho(0)$ is the ratio of the even to odd mode characteristic impedance at $x=0$. That is,

$$\rho(0) = z_{0e}(0)/z_{0o}(0). \tag{28}$$

It should be noted that, from the realizability condition (20),

$$\rho(0) \geqq 1, \tag{29}$$

or generally

$$\rho(x) = z_{0e}(x)/z_{0o}(x) \geqq 1 \qquad (0 \leqq x \leqq l). \tag{30}$$

Equation (26) offers the equivalence of the coupled nonuniform transmission-line folded all-pass network and the single nonuniform transmission-line resonator of characteristic impedance $z_{0e}(x)$ with the far end open circuited; that is, the reflected wave of the open-circuited single nonuniform transmission-line resonator corresponds to the transmitted wave of the folded all-pass network (see Fig. 6). Making use of this equivalence allows coupled nonuniform transmission-line folded all-pass networks to be synthesized by means of the methods now available for single nonuniform transmission-line resonators [14]–[16]. Thus, the problem of a coupled transmission line reduces to that of a single transmission line.

As long as we treat single nonuniform transmission lines, all-pass properties cannot be realized; however, this can be

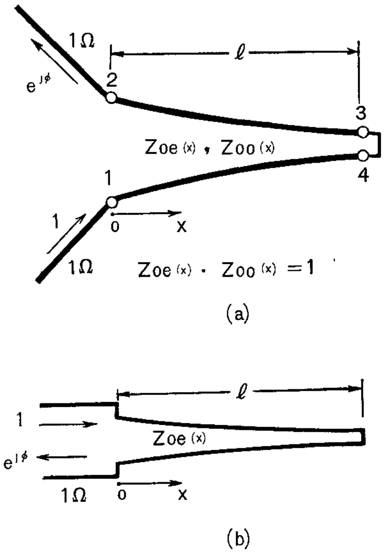

Fig. 6. Equivalence of a coupled nonuniform transmission-line folded all-pass network and a open-circuited single nonuniform transmission-line resonator (ϕ: phase shift).

done by the use of coupled nonuniform transmission lines as described before. In particular, the folded network treated in this section not only is an all-pass network but also possesses such a peculiar phase or delay characteristic that may be considered useful in many UHF and microwave systems that require phase shaping or delay equalization. Of course, the synthesis of coupled nonuniform transmission-line folded networks is complicated compared with the case of coupled uniform transmission lines. However, Youla's synthesis method [16] for arbitrarily terminated single nonuniform transmission lines is directly applicable because of the analytical equivalence shown in Fig. 6, and the peculiar properties not attainable by the stepped design using conventional coupled uniform transmission lines may be realized from its nonuniformity. The practical advantage gained by the use of nonuniform transmission-line folded networks is that the discontinuity effect of the physical junctions between adjacent coupling sections is eliminated. Coupled nonuniform transmission-line folded networks will be compared with coupled uniform transmission-line folded networks in Section IV.

B. Coupled Nonuniform Transmission-Line Directional Couplers

Consider the coupled nonuniform transmission-line four-port terminated by unit impedances at every port as shown in Fig. 7. Its behavior may be analyzed by the method of Reed and Wheeler [3]. When two signals of half amplitude and in-phase are applied at Ports 1 and 2, the plane of symmetry may be replaced by a magnetic wall, i.e., even mode case. Similarly, when two signals of half amplitude and out-of-phase are applied at Ports 1 and 2, the plane of symmetry may be replaced by an electric wall, i.e., odd mode

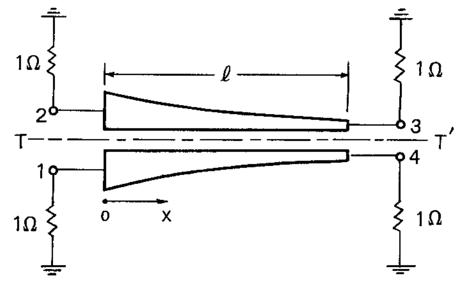

Fig. 7. Coupled nonuniform transmission-line directional coupler.

case. In each case, the problem reduces to that of a two-port network, and the sum of these two cases is a single signal of unit amplitude applied to Port 1. The amplitude and phase of the signals emerging from the four ports are given by

$$\begin{aligned} A_1 &= (\Gamma_e + \Gamma_o)/2 \\ A_2 &= (\Gamma_e - \Gamma_o)/2 \\ A_3 &= (T_e - T_o)/2 \\ A_4 &= (T_e + T_o)/2, \end{aligned} \tag{31}$$

where Γ_e and Γ_o are the reflected waves, and T_e and T_o are the transmitted waves, for the even and odd mode half section single nonuniform transmission-line two-port networks, respectively. These reflected and transmitted waves can be related to the two-port transfer matrix parameters of the even and odd mode half sections of the coupled nonuniform transmission-line four-port by

$$\begin{aligned} \Gamma_{\left\{\substack{e\\o}\right.} &= \frac{A_{\left\{\substack{e\\o}\right.} + B_{\left\{\substack{e\\o}\right.} - C_{\left\{\substack{e\\o}\right.} - D_{\left\{\substack{e\\o}\right.}}{A_{\left\{\substack{e\\o}\right.} + B_{\left\{\substack{e\\o}\right.} + C_{\left\{\substack{e\\o}\right.} + D_{\left\{\substack{e\\o}\right.}} \\ T_{\left\{\substack{e\\o}\right.} &= \frac{2}{A_{\left\{\substack{e\\o}\right.} + B_{\left\{\substack{e\\o}\right.} + C_{\left\{\substack{e\\o}\right.} + D_{\left\{\substack{e\\o}\right.}} \cdot \end{aligned} \tag{32}$$

Noting that from (22) and (32),

$$\begin{aligned} \Gamma_e &= -\Gamma_o \\ T_e &= T_o, \end{aligned} \tag{33}$$

we find

$$\begin{aligned} A_1 &= 0 \\ A_2 &= \Gamma_e \\ A_3 &= 0 \\ A_4 &= T_e. \end{aligned} \tag{34}$$

Equation (34) shows that, under the condition (21), the coupled nonuniform transmission-line four-port terminated by unit impedances behaves as a directional coupler perfectly matched and isolated at all frequencies and, in addition, if the functional forms of the even and odd mode characteristic impedances are given, the coupling to Port 2 may be found from the following equation

$$A_2 = \frac{m_1{}^e - j\beta \cdot m_6{}^e \cdot z_{0e}(l) - m_2{}^e/j\beta \cdot z_{0e}(0) - m_6{}^e \cdot z_{0e}(l)/z_{0e}(0)}{m_1{}^e - j\beta \cdot m_6{}^e \cdot z_{0e}(l) + m_2{}^e/j\beta \cdot z_{0e}(0) + m_5{}^e \cdot z_{0e}(l)/z_{0e}(0)}, \tag{35}$$

where the m's are given by (18). Equation (34) also offers the equivalence [17] (see Fig. 8) of the coupled nonuniform transmission-line directional coupler and the single nonuniform transmission-line section of characteristic impedance $z_{0e}(x)$. That is, the reflected wave of the single nonuniform transmission line corresponds to the backward-coupled wave of the directional coupler, and the transmitted wave of the single nonuniform transmission line corresponds to the forward-coupled wave of the directional coupler. The use of this equivalence reduces the synthesis of coupled nonuniform transmission-line directional couplers to that of single nonuniform transmission lines. Let us now consider this problem briefly. The coupling to Port 2 of the asymmetrical n-section coupled uniform transmission-line directional coupler having Chebyshev response is [18],

$$|A_2|^2_{\text{uniform}} = \frac{c^2 - h^2 \cdot T_n^2\left(\cos\frac{\beta l}{n} \Big/ \cos\frac{\beta_0 l}{n}\right)}{1 + c^2 - h^2 \cdot T_n^2\left(\cos\frac{\beta l}{n} \Big/ \cos\frac{\beta_0 l}{n}\right)}, \qquad (36)$$

where T_n is the Chebyshev polynomial of the first kind of degree n, c, and h are constants, β_0 is the phase constant at the lower equal-ripple band-edge frequency, and l is the total length of the coupler. Allowing the number of sections to increase indefinitely for a fixed overall length, the asymmetrical coupled nonuniform transmission-line Chebyshev coupler results, for which the coupling is

$$|A_2|^2_{\text{nonuniform}} = \frac{c^2 - h^2 \cdot \cos^2(l\sqrt{\beta^2 - \beta_0^2})}{1 + c^2 - h^2 \cdot \cos^2(l\sqrt{\beta^2 - \beta_0^2})}, \qquad (37)$$

since [19],

$$\lim_{n \to \infty} T_n\left(\cos\frac{\beta l}{n} \Big/ \cos\frac{\beta_0 l}{n}\right) = \cos(l\sqrt{\beta^2 - \beta_0^2}), \qquad (38)$$

which is the limiting form of the Chebyshev polynomial as its degree increases without limit. Thus, the synthesis of the coupled nonuniform transmission-line Chebyshev coupler reduces to that of the single nonuniform transmission-line section having reflection characteristic given by (37). For loose couplers, synthesis may be performed by the usual Fourier transform method for tapered impedance matching sections [19]–[21]; however, for tight couplers, this method is not directly applicable, and the higher-order theory [16], [22] must be used.

If we select a variation of the even and odd mode characteristic impedances of the types

$$z_{0\{^e_o}(x) = z_{0\{^e_o}(l - x), \qquad (39)$$

then a symmetrical coupled nonuniform transmission-line directional coupler [23] can be obtained, for which the phase difference between the two outputs at Ports 2 and 4 is 90-degrees at all frequencies.[3] It is this property that makes

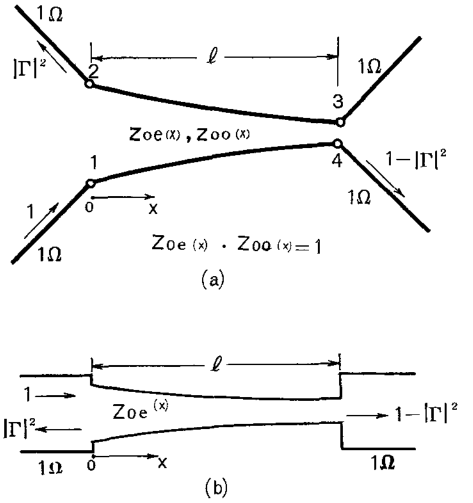

Fig. 8. Equivalence of a coupled nonuniform transmission-line directional coupler and a single nonuniform transmission line.

symmetrical couplers [24], [25] of importance.

Comparing with familiar multi-section coupled uniform transmission-line couplers, both symmetrical and asymmetrical coupled nonuniform transmission-line couplers possess no discontinuity in the coupling region, and therefore should be capable of providing higher isolation and better input match. Furthermore, they offer the possibility of the realization of ultra-broadband couplers because of their nonuniformities. If there need not be any particular phase relationship between the outputs, asymmetrical couplers seem to be superior to symmetrical ones since the former offers the smaller size. However, as in the case of asymmetrical uniform transmission-line couplers [18], a major practical disadvantage of symmetrical nonuniform transmission-line couplers is the presence of the abrupt discontinuity at one of the two ends, which may cause considerable degradation of performance.

IV. Coupled Exponential Line

In Sections II and III, we have considered the general coupled nonuniform transmission line, not assuming any specific type of variation of the characteristic impedances. For the purpose of illustration, let us now investigate the properties of the coupled nonuniform transmission-line networks, treated in the previous section, using the coupled exponential line as the network element, of which even and odd mode characteristic impedances (normalized) vary exponentially along the longitudinal direction; that is,

$$z_{0e}(x) = z_{0e0} \cdot \exp(\mu x)$$
$$z_{0o}(x) = z_{0o0} \cdot \exp(-\mu x) \qquad (0 \leq x \leq l), \qquad (40)$$

where $z_{0\{^e_o}$ are the even and odd mode characteristic impedance levels of the line at $x = 0$, and, from the condition (21), are related by

$$z_{0e0} \cdot z_{0o0} = 1. \qquad (41)$$

The rate of taper μ in (40) may be positive or negative; how-

[3] Although the details of the synthesis procedure have not yet been presented, such symmetrical nonuniform transmission-line couplers have recently been treated by Tresselt [23].

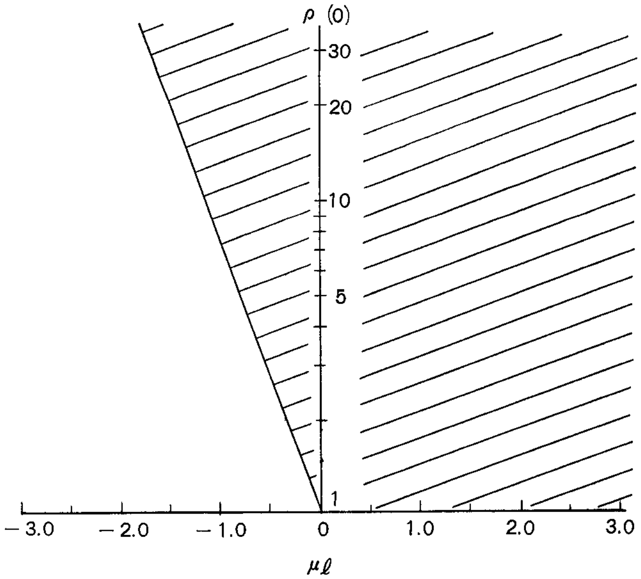

Fig. 9. Limitation on parameters, $\rho(0)$ and μl, for the coupled exponential line defined by (40) and (41).

ever, it is necessary, so as to satisfy the realizability condition (20), to choose the parameters μ and ρ $(0)(=z_{0e}(0)/z_{0o}(0))$ so that

$$\rho(0) \geqq 1 \qquad (\mu \geqq 0)$$
$$\rho(0) \geqq \exp(-2\mu l) \qquad (\mu < 0). \tag{42}$$

The shaded region in Fig. 9 represents the range of the parameters for which the coupled exponential line is realized.

For convenience of later discussion, the two-port transfer matrix parameters of the even and odd mode half sections of the coupled exponential line defined by (40) and (41) will now be derived.

$$A_e = D_o = \frac{1}{P\cdot q}\sin\Theta\{p + q\cdot\cot\Theta\}$$
$$B_e = C_o = j\frac{z_{0e0}\cdot P}{q}\sin\Theta$$
$$C_e = B_o = j\frac{1}{z_{0e0}\cdot P\cdot q}\sin\Theta$$
$$D_e = A_o = \frac{P}{q}\sin\Theta\{-p + q\cdot\cot\Theta\}, \tag{43}$$

where,

$$P = \exp\left(\frac{\mu l}{2}\right)$$
$$p = \frac{\mu}{2\beta}$$
$$q = \sqrt{1 - p^2}$$
$$\Theta = \beta\cdot q\cdot l. \tag{44}$$

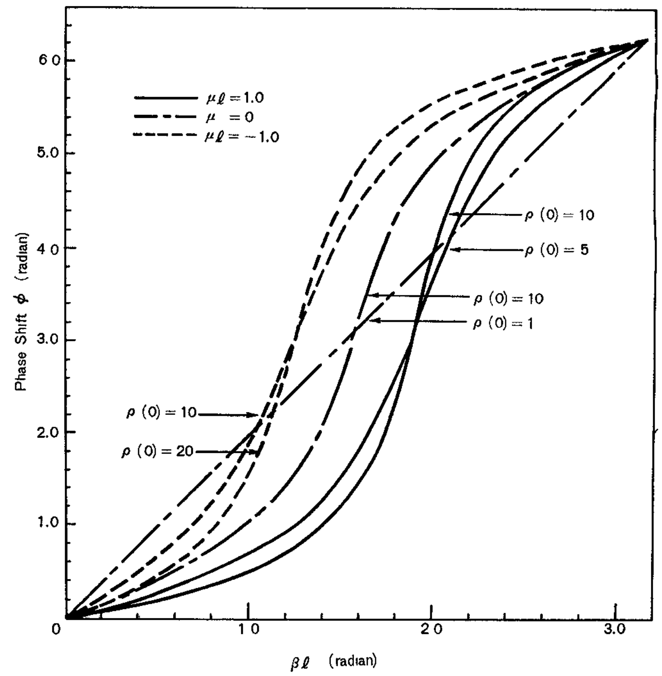

Fig. 10. Phase characteristics of the coupled exponential-line folded all-pass networks.

A. Coupled Exponential-Line Folded All-Pass Networks

The phase shift ϕ through the coupled exponential line folded all-pass network as in Fig. 5 is

$$\phi = 2\cdot\tan^{-1}\left[\frac{1}{\sqrt{\rho(0)}\,(p + q\cdot\cot\Theta)}\right]. \tag{45}$$

Curves are plotted in Fig. 10, showing Φ against βl. Inspection of Fig. 10 shows that this network possesses peculiar phase characteristics. Also, in Fig. 10, the $\phi-\beta l$ curves with $\mu=0$, in fact, correspond to those of coupled *uniform* transmission-line folded all-pass networks. It should be noted that, as the rate of taper μ increases from the negative value through zero to the positive one, the $\phi-\beta l$ curve for the coupled exponential-line folded all-pass network is shifted from the left to the right for a *constant* length of the line, and the variation in maximum slope of the $\phi-\beta l$ curve is accomplished by varying $\rho(0)$, i.e., the ratio of even to odd mode characteristic impedance at $x=0$. This is the property that makes coupled exponential-line (or generally coupled nonuniform transmission line) folded networks so interesting and useful. This shifting property cannot be obtained by using coupled uniform line folded networks for constant length of the line, since the $\phi-\beta l$ curves for $\mu=0$ in Fig. 10 pass through the point ($\beta l=\pi/2$, $\phi=\pi$) independent of $\rho(0)$. In order to shift the $\phi-\beta l$ curves with uniform transmission lines, cascaded folded networks [13] must be employed. However, this means the degradation of performance because of its junction effect and furthermore the size of the whole network becomes larger.

Next we shall consider the delay characteristics of coupled exponential-line folded all-pass networks. The delay versus frequency function is by definition,

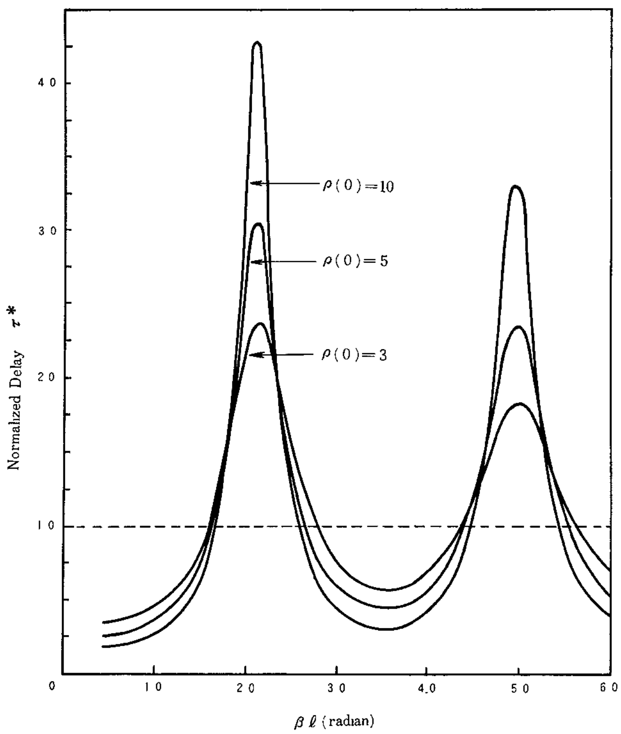

Fig. 11. Delay characteristics of the coupled exponential-line folded all-pass networks ($\mu l = 1.5$).

$$\tau = \frac{d\phi}{d\omega}, \tag{46}$$

where ω is the angular frequency. Let the normalized delay function τ^* be defined as

$$\tau^* = \frac{\tau}{\tau_0}. \tag{47}$$

Here τ_0 is the delay produced when TEM wave propagates along a *single uniform* transmission line of length $2l$ (twice the length of the folded network), and is given by

$$\tau_0 = \frac{2l}{v}, \tag{48}$$

where v is the velocity of propagation of TEM wave. Then we get from (45)–(48)

$$\tau^* = \frac{1}{2l}\frac{d\phi}{d\beta} = \sqrt{\rho(0)}\,\frac{(1+\cot^2\Theta) + p(q - p\cdot\cot\Theta)/\Theta}{1+\rho(0)\cdot(p + q\cdot\cot\Theta)^2}. \tag{49}$$

Delay characteristics of the coupled exponential-line folded all-pass networks are illustrated in Fig. 11 for $\mu l = 1.5$, where the dotted line shows (48); that is, delay obtained by the single uniform transmission line of length $2l$. Inspection of Fig. 11 shows that, by using the folded network, larger delay can be obtained at the frequencies near the peak position than is obtainable with the single uniform delay line of the same overall length. Variation in peak height is accomplished by varying $\rho(0)$, and variation in peak position by varying the rate of taper. Therefore, if the proper selection of the functional forms of the characteristic impedances and the parameters is made, folded all-pass networks may be used as delay equalizers for various applications, e.g., for use in wideband PCM transmission [26]. The main advantage gained by the use of the folded equalizers is that there is no need of a circulator which is necessary in order to separate the input and output waves for the usual reflection type delay equalizers.[4]

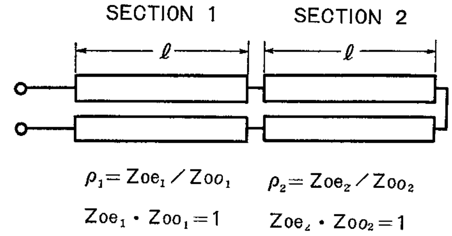

Fig. 12. Cascaded two-section coupled uniform transmission-line folded all-pass network.

If we connect in tandem folded networks with the same peak height but slightly different peak positions, then it is possible to obtain an approximately flat delay response, and the overall network may be used as a constant delay network. In comparison with the usual single uniform delay line, the folded delay network becomes extremely compact because of the folding process and larger delay near the peak position.

Taking the coupled exponential line as representative of coupled nonuniform transmission lines, let us now compare coupled nonuniform transmission-line folded networks with uniform ones. For the coupled uniform transmission-line folded network, the peak position of the delay characteristic is fixed at $\beta l = \pi/2$, since its one parameter ρ (the ratio of even to odd mode characteristic impedance) permits only the variation in peak height; the coupled exponential-line folded network, on the other hand, permits the variation in both peak position and peak height because of its two parameters μ and $\rho(0)$. As a particularly simple example, consider the design of the delay network having maximum delay at $f = 1000$ MHz ($\lambda = 30$ cm). If we use a coupled uniform transmission-line folded network, the required length is 7.5 cm. On the other hand, if we choose the coupled exponential-line folded network with negative μ,[5] having peak position at, for example, $\beta l = \frac{3}{8}\pi$, the required length is 5.6 cm. Thus, a reduction in length is realized. With uniform transmission lines, variation in both peak height and peak position is accomplished if we employ the cascaded two-section folded network having two parameters as in Fig. 12

[4] It is interesting to note the relation of the reflection type equalizer to the single transmission-line equivalent network of the folded equalizer shown in Fig. 6(b). In the design of the folded equalizer, synthesis may be performed by the method for the reflection type equalizers, but it is realized in the form of a folded network which enables the designer to avoid the use of a circulator.

[5] Since the (group) delay is proportional to the slope of the $\phi - \beta l$ curve from its definition, it is seen from Fig. 10 that the coupled exponential-line folded network with positive μ produces maximum delay at $\beta l (> \pi/2)$, whereas that with negative μ produces maximum delay at $\beta l (< \pi/2)$.

(see Steenaart [26], Fig. 5). However, as previously described, the discontinuity effect may degrade the performance and the size becomes larger. Of course, generally speaking, the previous discussion seems insufficient to be conclusive since only the coupled exponential line is treated in this paper; however, even from such a simple example, it is seen that the coupled nonuniform transmission-line folded networks are worthy of mention.

B. Coupled Exponential-Line Directional Couplers

Coupling to Port 2 of the asymmetrical coupled exponential-line directional coupler, as shown in Fig. 7, is derived from (32), (34), and (43) as

$$A_2 = \frac{\left\{-q\cdot\cot\Theta\cdot\sinh\frac{\mu l}{2} + p\cdot\cosh\frac{\mu l}{2}\right\} + j\,\dfrac{k\left(\frac{l}{2}\right)}{\sqrt{1-k\left(\frac{l}{2}\right)^2}}}{\left\{q\cdot\cot\Theta\cdot\cosh\frac{\mu l}{2} - p\cdot\sinh\frac{\mu l}{2}\right\} + j\,\dfrac{1}{\sqrt{1-k\left(\frac{l}{2}\right)^2}}}, \tag{50}$$

where $k(l/2)$ is the coupling factor at $x=l/2$ and is defined by

$$k\left(\frac{l}{2}\right) = \frac{z_{0e}\left(\frac{l}{2}\right) - z_{0o}\left(\frac{l}{2}\right)}{z_{0e}\left(\frac{l}{2}\right) + z_{0o}\left(\frac{l}{2}\right)}, \tag{51}$$

or generally the coupling factor is

$$k(x) = \frac{z_{0e}(x) - z_{0o}(x)}{z_{0e}(x) + z_{0o}(x)} \qquad (0 \leqq x \leqq l). \tag{52}$$

Let us now consider the special case where $k(0)=0$, for which (50) reduces to

$$A_2 = \frac{\left\{-q\cdot\cot\Theta + p\cdot\coth\frac{\mu l}{2}\right\} + j\cdot 1}{\left\{q\cdot\cot\Theta\cdot\coth\frac{\mu l}{2} - p\right\} + j\cdot\coth\frac{\mu l}{2}}. \tag{53}$$

As the frequency tends to infinity, the amplitude of A_2 approaches constant. This asymptotic value corresponds to the mean coupling, C (dB), of the asymmetrical coupled exponential-line directional coupler; that is,

$$C = \lim_{\beta\to\infty} 20\cdot\log_{10}\frac{1}{|A_2|} = 20\cdot\log_{10}\coth\frac{\mu l}{2}\,. \tag{54}$$

Curves are plotted in Fig. 13, showing coupling characteristics for several mean coupling values. It can be seen that, in comparison with the *bandpass* characteristics of the usual symmetrical [24], [25] or asymmetrical [18] multi-section coupled uniform transmission-line couplers, asymmetrical coupled exponential-line couplers possess the *high-pass* characteristics. Thus, the spurious response unavoidable with cascaded uniform transmission-line couplers is eliminated. The steps required to design the asymmetrical coupled exponential-line couplers may be summarized as follows:

1) From the desired value of mean coupling, C(dB), obtain μl by the use of (54).
2) Using Fig. 13, determine the total length of the directional coupler from the values of the allowable coupling deviation and the required lower band-edge frequency.
3) Find the rate of taper μ.

It should be noted that the *high-pass* characteristic of the asymmetrical coupled exponential-line directional couplers is based on the *end condition* $k(0)=0$. As long as conventional coupled line configurations are employed, it is impossible to realize this condition which means that the coupled lines must be infinitely far apart. However, the use of the slit-coupled configurations shown in Fig. 14 allows this condition to be realized with reasonable coupled line spacing, for which exact design equations have been given by the present authors.[6] By varying the slit width, both configurations in Fig. 14 permit smooth variation in coupling with constant strip spacing and thus these are the suitable configurations for coupled nonuniform transmission lines. In the uniform or nonuniform transmission-line directional couplers using slit-coupled configurations, in order to avoid undesirable coupling, the slit width is set equal to zero in the terminating region. Therefore, it is found that the end condition $k(0)=0$ for the asymmetrical coupled exponential-line directional couplers can be easily realized by using the slit-coupled configurations.[7] In other words, this condition would tend to permit the smooth transition from the terminating region to the coupling region.

Theoretically, asymmetrical coupled exponential-line directional couplers possess excellent characteristics; however, as previously described, a practical disadvantage of such couplers is the presence of the abrupt discontinuity at one end, as in the case of asymmetrical multi-section coupled

[6] S. Yamamoto, T. Azakami, and K. Itakura, "Slit-coupled strip transmission lines," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-14, pp. 542–553, November 1966.

[7] It should be also emphasized that the slit-coupled configurations are available for the realization of the folded networks having decreasing $\rho(x)$ with increasing x (for example, coupled exponential line with negative μ), since they permit the variation in coupling with constant coupled line spacing, for which conventional coupled line configurations are not suitable.

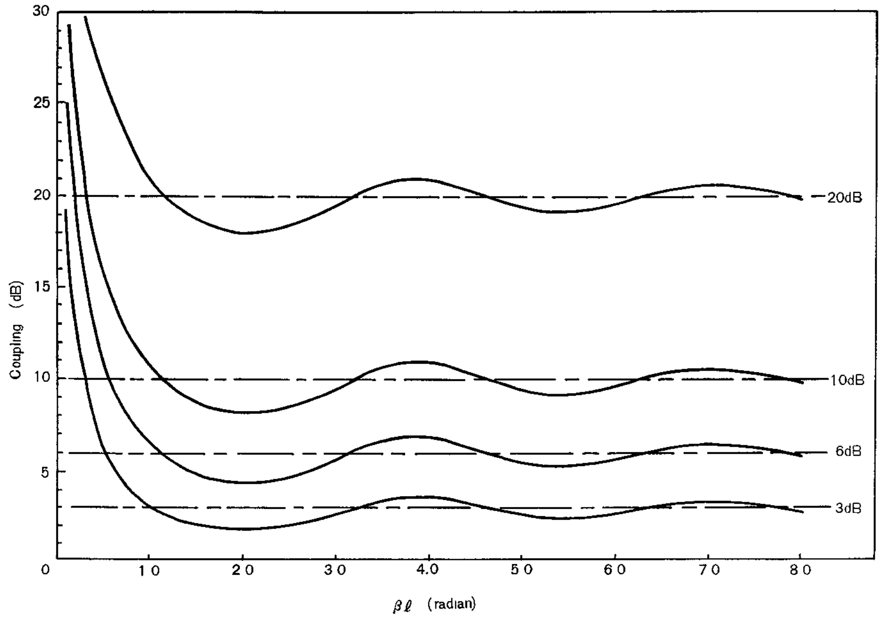

Fig. 13. Coupling characteristics of the asymmetrical coupled exponential-line directional couplers ($k(0)=0$).

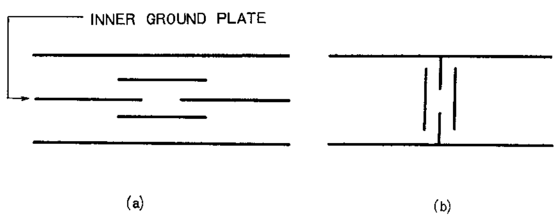

Fig. 14. Cross-sectional views of the slit-coupled strip-line configurations: (a) parallel case, (b) perpendicular case.

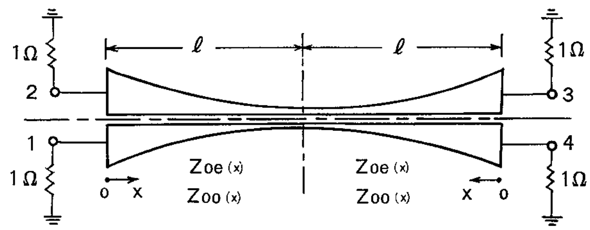

Fig. 15. Two-section symmetrical coupled nonuniform transmission-line directional coupler.

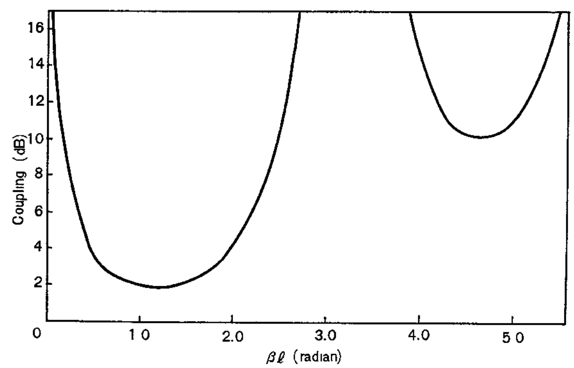

Fig. 16. Coupling characteristic of the two-section symmetrical coupled exponential-line directional coupler ($k(0)=0$, $\mu l=1.5$).

uniform transmission-line couplers [18]. The use of symmetrical couplers such as those described by Tresselt [23] allows such a discontinuity to be avoided, since the strongest coupling region is in the center and not at one end; however, a high-pass characteristic is then not attainable. If we connect two identical asymmetrical coupled exponential-line couplers in cascade as in Fig. 15, the symmetrical coupler results; however, calculation shows that such a simple symmetrical coupler possesses poor bandpass characteristic not sufficient for most applications (see Fig. 16). Therefore we find that, in order to obtain the characteristic impedance functions giving broadband symmetrical coupled nonuniform transmission-line directional couplers, synthesis must be performed as was done by Tresselt.

V. Conclusions

Coupled nonuniform transmission lines have been shown to be useful distributed network elements. We have seen that it is possible to describe the behavior of coupled nonuniform transmission lines in a very concise and compact way so that matrix parameters may be derived by solving linear second-order differential equations for more conventional single nonuniform transmission lines. Various matrix representations of a general coupled nonuniform transmission line have been presented, each of which may serve as a basis for the analysis and design of coupled nonuniform transmission-line networks. As specific applications, the coupled nonuniform transmission-line folded all-pass networks and the coupled nonuniform transmission-line directional couplers have been treated in detail, and useful equivalences have been presented, which allow the syntheses of these networks to be performed by using single nonuniform transmission-line techniques. In addition, the properties of these two networks using the coupled exponential line have been investigated.

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